Teacher Work Sample

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Mathematics and Statistics Education

March 30, 2018
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Learning Context

School District: InTech Collegiate High School District

Name of School: InTech Collegiate High School (ICHS)

Title 1 School: Yes

Demographics of School: There are a total of 179 students at InTech Collegiate High School. 36.3% are female, 63.7% are male. Of those students there are 74.3% White, 14% Hispanic, 6.7% Asian, 2.2% Black, 1.7% Multiple Races, and 1.1% American Indian or Alaskan Native. Approximately 6.7% of students are gifted, 25.7% of students have either an IEP or 504, and 2.8% are English Language Learners. According to SAGE results, in the 2016-2017 school year InTech Collegiate High School was 85% proficient in mathematics, 74% proficient in language arts, and 63% proficient in science. Which is a 5% increase in mathematics, but 6% decrease in language arts, and a 11% decrease in science from the 2015-2016 school year.

Description of school climate [who attends, leadership style, parent/community involvement, school-wide discipline plan (if any), physical environment, academic environment]: InTech Colligate High is located in North Logan on the Utah State University Innovation Campus right next to the Space Dynamics Lab. ICHS is made up of students from all over Cache Valley, and from a mix of economic backgrounds. InTech is a charter high school so students apply to attend and are selected through a lottery method. Jason Stanger is the principal and due to the small size of the student body he is able to be very hands-on and gets to know the students personally. Parents have the opportunity to participate in and serve on the ICHS Governing Board and the Parent Involvement Council where they help provide mission and policy level direction to the school. ICHS also has many parent volunteers who help with
activities around the school like judging the science fairs. Students who attend InTech are expected to follow the school-wide discipline plan which is outlined in the ICHS Student Handbook on page 45 which states that students are expected to follow accepted rules of conduct, show respect for other people, and obey persons in authority at the school. Students are reminded of their expectations periodically throughout the school year either in announcements, or assemblies. Because InTech is a STEM-based, early college charter high school, most of the students who attend are interested in the STEM fields and academia. Therefore the academic environment supports excellence in the sciences, technology, engineering, and mathematical fields, and students who are eligible, usually take classes straight from Utah State University while still in high school.

**Grade level:** 10th-12th

**Learning environment [attendance, classroom management plan, seating arrangement, level of student engagement in learning, level of safety for learning]:** Approximately 97% of the 11 students in my focus class attend daily. The classroom is very long and narrow with the white board on the longer wall. This makes setting up an accessible classroom very challenging so my cooperating teacher has set two rows of desks near the “front” facing the white board, then there is an aisle, then two more rows of desks in the “back.” This allows for the teacher to freely walk in between desks and be able to easily see what students are doing. This is a great help for classroom management as it is easy to make it to anywhere in the classroom in case of needed assistance. It also helps students be engaged in learning as it is easy for them to see the board and they are close enough to their neighbors to collaborate in groups of four when needed. Two of the major rules in the classroom are to ask help if you need it, and to keep trying. These rules
have really helped in increase student engagement/involvement as they feel safe to ask questions or answer them. There is absolutely no tolerance for bullying or belittling.

**Subject matter of lessons:** Trigonometry Functions

**Total number of students:** 11

**Students with special needs and short explanation of the needs:**

**With IEPs:** I have a total of two students with IEPs in my classes, and one of them is in my focus class. This student requires the assistance of technology in the form of a laptop and is able to type notes, assignments, and tests. This student also needs clear, succinct, instructions, written as well as oral instructions, and whenever possible to limit copying activities and provide copies of class or lecture notes. This student also will be allowed extra time on assignments, tests, and projects.

**Students who receive speech/language services:** None of the students in my classes receive speech/language services.

**English language learners:** There are a total of 5 of these students in the school, but none of which are in my classes.

**Gifted and talented:** There are a total of 12 of these students in the school, but none of which are in my classes.

**Other (e.g., 504 plans—please specify):** I have a total of seven students with 504 plans in my classes, and one of which is in my focus class. This student needs extra time on assignments and tests, to sit in the front of the classroom near the teacher, and due to medication, may need to eat a snack during class.
Students’ prior knowledge for these lessons: All students in my focus class have completed Secondary Math 2 and some have completed and intermediate math class called Mathematic Decision Making for Life. In both of these courses, students are introduced to trigonometry but it was mostly in relation to solving right triangles. In the unit we are going to cover, students will be learning about trigonometric functions. We made sure to revisit transformations of parent functions and writing functions that model an observation. Students were also given a Trigonometric Functions Pre-Test where it became apparent that some students were able to recall their previous knowledge of trigonometric functions but the majority did not and only a very select few knew how trigonometric functions related to the unit circle.

Students’ background and interests for these lessons: In the past, the students have been very receptive to lessons with hands-on activities. Students are not only more interested, but they seem to remember the lessons with activities better and therefore they remember the material taught better as well. My cooperating teacher has done one of these activities (see lesson 9.1.1) in the past and has hung the finished product on the wall in her room. Students have commented on it and are looking forward to participating in this lesson in class.

How did your knowledge of these students and assessment of their prior knowledge inform your lesson planning? The students here at InTech chose to be here, and for most of them, it is because they are interested in the STEM fields which are heavily influenced by discovery, getting to know how things work, and why they work. This describes many students in my focus class and after learning more about these students and their prior knowledge; I have adjusted my lesson planning according to their needs and interests. I included some hands-on activities in my lessons as an aid to help students understand how mathematics models the world around us, and to help increase the participation from a few students which are typically not as highly engaged. I
have a couple students who need succinct and clear instructions and guided notes, so I provide my students with access to the lecture notes before class and access to the completed lecture notes afterwards. Also many of my students find one-on-one instruction very helpful in addressing their individual questions and concerns, so I have provided both in-class time where students will do their daily quizzes and work on assignments. During this time, students are free to ask me questions and receive that one-on-one instruction. These lessons also build upon each other so I frequently review material learned in previous class periods both during lectures and with daily quizzes.
Focus Students

**Description of Student 1**

**Prior learning:** Student 1 did very poorly on the Trigonometric Functions Pre-test. In Secondary Math 2, which Student 1 took, students are taught how to use trigonometric functions to help solve right triangles. On the pre-test, there were two questions involving using trigonometric functions and a right triangle to identify the coordinates of a point on a unit circle. Student 1 was unable to correctly identify the trigonometric functions needed, and therefore the coordinates of the point. The last question on the pre-test was a writing prompt to which the students were to briefly describe their knowledge of trigonometric functions. In response to this prompt Student 1 wrote, “I know pretty much nothing.”

**Academic ability:** It has been fairly clear from early on that mathematics is not Student 1’s strong suit, and he requires extra support. While taking Secondary Math 1 and 2 he earned mostly C letter grades, so it was decided that he should be placed in an intermediate class between Secondary Math 2 and Secondary Math 3. Here at InTech this class is called Mathematical Decision Making for Life and incorporates material the student should have learned in Secondary Math 2, some mathematics commonly used in everyday life, and an introduction to some material the student learns in greater depth when he takes Secondary Math 3. In 2016, Student 1 earned a 3 on the Secondary Math 2 SAGE test, which is an improvement from his score of a 2 in 2015 on the Secondary Math 1 SAGE test. Student 1 also earned 2’s on his SAGE Language arts and SAGE Physics tests in 2015 and 2016.

**Personal background:** Student 1 is a senior here at InTech and has been here for 9th through 12th grades. As far as I am aware his family life is stable and agreeable. His parents are
married, and I believe his father is in the construction business. An interest in building things must have been instilled into Student 1 from a young age because he is on our First Robotics team here at InTech which builds robots and competes in regional and national competitions. He is white and English is his primary language.

**Other relevant characteristics:** Student 1 has learned that he needs extra time and support in order to retain information. He has the desire to succeed and is willing to put in the effort to learn. I have also noticed that Student 1 likes to see the overall picture of mathematics and likes to know how what we are learning connects to what he already knows.

**Influence of all of these characteristics on your teaching:** In order to help Student 1 more effectively, I have structured my classes and overall schedule to allow for time to help him and other students more one-on-one. This allows him to ask questions on aspects of the material that are confusing or that he does not fully understand. I also provide students access to the guided lecture notes beforehand and copies of the completed lecture notes after the lecture. Student 1 takes advantage of these notes as it allows for him to focus more on the mathematics and mathematical thinking of the material we are learning and focus less on trying to copy examples down.

**Description of Student 2**

**Prior learning:** Student 2 performed relatively poorly on the Trigonometric Functions Pre-test, but in comparison to the overall performance of the class, she received an average score. She was able to correctly identify the graph of the sin(x) function and to use trigonometric functions to approximate the coordinates of a point on a unit circle using a right triangle. This is
material Student 2 learned in Secondary Math 2 which she took during the 2016-2017 school year.

**Academic ability:** She has received a high portion of A’s and B’s in all her classes but mainly B’s in her mathematics courses. She has earned 4s on all of her SAGE Math and SAGE Science test and 3s on her SAGE Language arts tests for the past three years. Student 2 participates in class discussions and usually helps explain material to other students who are having a hard time understanding. Student 2 does well on assignments and daily quizzes but sometimes has a hard time turning things in on time. She also seems to pick and choose which assignments she will turn in.

**Personal background:** Student 2 is a sophomore here at InTech and started attending here her freshman year. As far as I am aware her family life is agreeable, and she has many friends here at InTech. I do not believe her parents are married, but she is allowed to be checked out by both of them so they have shared custody. Student 2 is also interested in engineering and is involved in our First Robotics team. She has taken almost two years of engineering courses here at InTech and has received high marks in them. She is Asian and English is her primary language.

**Other relevant characteristics:** During lessons, student 2 will observe and take notes when a new topic is being introduced, but as she becomes more familiar with it, she is confident to ask questions, answer others, and participate in discussions. Once she is given the “tools” she needs, she is usually able display competence in the material area.

**Influence of all of these characteristics on your teaching:** In order to help Student 2 more effectively, I have incorporated a lot more discussions into my lesson plans. Since we have
smaller class sizes compared to a public high school, it is easier to have class discussions about material we are learning and give everyone the opportunity to express what they are thinking. Student 2 usually participates in these discussions and by the end is answering other students’ questions. I have also tried to demonstrate examples in small easy steps and explain things while I present them on the board. This is in an effort to help students, including Student 2, to connect what they are learning to things they have already learned and to understand the “rules” of mathematics. In an effort to encourage Student 2 to turn in all her assignments, I will periodically give verbal reminders of assignments and their due dates in addition to the daily reminders she receives on Canvas.
Graphing the Sine Wave via the Screamer

(Based off of Core Connections Integrated III, Second Edition, Chapter 9.1.2 by Judy Kysh and Michael Kassarjian)

Subject and grade level: Secondary Math 3- Grades 10-12

Approximate time: 65 minutes

Rationale for Methods: This lesson follows a Discover-the-Relationship type lesson (also called inquiry-based learning) with the aid of a hands-on activity using a model of “The Screamer.” The Discover-the-Relationship method is rooted in research of cognitive development which has shown that as students discover a mathematical relationship themselves, they are: (1) more likely going to remember it, (2) create a firm understanding of that topic, and (3) make more sense of other things concerning that topic, for example algorithms.

Content Standards:

• F.BF.: Write a function that describes a relationship between two quantities.
• F.IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
• F.IF.7e: Graph exponential and logarithmic functions showing intercepts and end behavior; and trigonometric functions, showing period, midline, and amplitude.
• F.TF.2: Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Academic Language/Vocabulary objectives:

• Language Skill: Students will need to collect data using a model and record any observations they have during the activity. Students will analyze and discuss patterns and connections between the data they collected and its graph. Students will then use their knowledge of trigonometry to discover the connection between the Unit Circle and the Sine graph.
• Discipline-Specific Vocabulary: Students will use the idea of The Screamer to be introduced to the topic of the Unit Circle and its properties. Students will then use their knowledge of angles (Theta) and trigonometry on right triangles to make the connection between the Unit Circle and the Sine Graph.

Required material, resources, and technology:

• Smart Board or some way to project “Graphing the Sine Wave via the Screamer” PowerPoint on the screen.
• Graphing Calculators, one per student
• Materials for activity:
Cardboard boxes (such as a shoe, cracker, or cereal box), one per team
- Box cutter
- Cardstock and transparency paper
- Toothpicks (or other objects that could be used as axles)

Prepare wheels to simulate The Screamer Ferris Wheel as follows:
- Lesson 9.1.2A Resource Page copied onto transparencies and cut out, one per team
- Lesson 9.1.2 B Resource Page copied onto cardstock and cut out, one per team.
- After they is printed and cut out, create “The Screamer” model by taking a toothpick and poking a hole first in the center of the Lesson 9.1.2A Resource Page (printed side towards you), then through the center of the Lesson 9.1.2B Resource Page (printed side towards you).
- To create the structure, take a box (one for each team) and cut a slit in it approximately 21 cm by 0.5 cm. Be sure that the box is deeper than 10 cm to allow room for the “underground” portion of the wheel.
- Place one of “The Screamer” models in each box by placing it through the slit and having the toothpick rest on the top of the box so it look like have of the wheel is inside the box and the other half is outside.
- Lesson 9.1.2C Resource Page, copied onto a transparency, cut into individual unit rulers, one per team.

The PowerPoint provides an outline of the lesson and students can print them out before hand so they will have a kind of guided notes on which to write down their observations. The Smart board allows for the instructor to record the students’ observations on the board and save them electronically for students to refer back to and also for students who were absent. The Screamer models allows for students to have a hands-on learning experience where they will discover how the unit circle relates to the sine wave. Lastly, the graphing calculators allow for each student to then input their observations recorded from working with The Screamer and graph them, thus helping them connect it to the sine wave.

**Lesson Objectives:** Students will use experimental data generated from measuring the heights of right triangles to create a sine graph (Standards F.BF, F.IF.4, F.IF.7e, and F.TF.2).

**Instructional Procedures:**

1. Project “Graphing the Sine Wave via the Screamer” PowerPoint on the electronic board/projector. Stand at front of classroom to cue to students the start of lecture and that they should pull out their notes.
2. Have a student read The Screamer problem off of the PowerPoint. Explain to students that they have been hired as the rescue team help get the people to safety. To help them in their efforts they will receive a model (a smaller version) of “The Screamer.” Students will need to measure (with their unit ruler) and record how high (or low) at least 15 of the
seats will be. Show students the PowerPoint slide with the table on it. Explain to them that in order to keep our observations in a neat and orderly fashion, we are going to use a chart with column and rows, in other words, a table. Instruct the students to make a similar table on their own sheet of paper.

3. Split students into groups of 2-3 and give each group a model of The Screamer with a unit ruler from Lesson 9.1.2C Resource Page. Students will measure, and make a table, of the heights of at least 15 different positions of the seats. Be sure that students understand that the transparency circle remains fixed and the cardstock circle turns. You may need to clarify the units; the 100-foot radius of “The Screamer” is a 10cm radius for the students’ model. Students will call this a length of 1 unit in order to create a unit circle.

4. After most of the groups have filled out their tables, recollect the class and have them report some of their findings. Write them on the board to fill out the table. Discuss with students any patterns they observed when measuring many different seat positions. Be sure to point out the symmetry in values after 90° and again after 180°.

5. Have students pull out their graphing calculators, enter in the table, and graph it. Have students use the “Degree of Rotation” as the x-axis and the “Height” as the y-axis.

6. Students should get a wave (the sine wave). Show them the PowerPoint slide with the graph of the sine function and discuss with students how we could have guessed what the graph would have looked like, referring back to the symmetry of the points.

7. Ask students to suppose they were asked to add 20 more data points to their table. What patterns did they notice that they could use to reduce the amount of work? Sample answers include: using the graph, the heights of the riders are the same on the top half of the ride as they are on the bottom half of the ride only negative.

8. Read Example 1 to the students and have them engage with it in their teams. For part a, they should come up with the equation \( \sin(40°) = \frac{x}{1} = x \) and this represents the height of riders who are 40° above ground.

9. Then instruct students to write an equation representing the escape height for any passenger. Have students use \( \theta \) to represent any given angle. Equation should be: \( \sin(\theta) = x \).

10. Have students pull out their graphing calculators and graph the data from their first two columns from their table they filled out in step 3. Then have them graph the function the created in step 7: \( \sin(\theta) = x \). Discuss with students what they observe.

11. Have students adjust their viewing window so they can see more of their graph. Discuss with students what they observe when \( \theta \) gets larger.

12. Lastly have students use the \([table]\) function on their calculator to see the values for their equation. Have the students add these values to their tables created in step 3 to the corresponding angles. What do they observe about the “heights” values? They are the same; therefore it models the sine function.

13. This relationship will be used in following lessons and Quizzes and will help provide a foundation on which to discover more properties of the unit circle and its connection with trig functions.
**Adaptations/Accommodations:** Using The Screamer model not only helps students to be more engaged and using different learning mediums, but it also allows Student 1 to see how something new he is learning, for example trig functions and the unit circle, connect to things he already knows like trig functions in relation to right triangles. Having students work in groups allows the instructor to walk around the room and listen to discussions and help students in smaller groups. This is a great help to Student 1 who then was able to ask questions and work with peers to answer questions and it provided Student 2 with the opportunity to discuss her findings and think critically about why she is observing what she is. Class discussions also provide Student 2 with an opportunity to share these discoveries. Also having printouts of the lesson PowerPoint allowed Student 1 to participate in class discussions instead of writing down material.

**Assessment:** Material learned in this lesson will be used in the next lesson to keep building knowledge of the unit circle and its connections to trig graphs. Students use a program called MyOpenMath to take quizzes in class during subsequent lessons on which this material will be formatively assessed. At the end of the unit there will also be a test where students will need to use their knowledge of the unit circle, sine function, and other trig functions to solve problems.
SECONDARY MATH 3

GRAPHING THE SINE WAVE VIA THE SCREAMER

Objective:
Students will use experimental data generated from measuring the heights of sight triangles to create a sine graph.

O.D.: M 1, P 1, F 1, A, M 2, 7, P 2, F 5, 1, P 2

THE SCREAMER

“HURRY! Let’s get there before the long lines go too long!” shouts Antonio to his best friend, Ron, as they race to get on The Screamer, the newest attraction at the local amusement park.

“Do only have open for one day, and already everyone is saying ‘the scariest ride of the park’” exclaims Antonio. “I hear they really had to rush to get it done in time for summer.”

Antonio watches as he races to a halt in front of the sign that says, “Welcome to The Screamer, the Screamer Rides on Earth.” The picture on the sign shows an enormous wheel that represents The Screamer, with its 100-foot radius. Half of the wheel is above ground level, in a very dark, nearly pitch of water at the bottom. As The Screamer rolls or revolving speeds, riders fly into the air before plunging downward through plumes of snow and ice and spindles that slice the air where they splash through the dark, eerie water on their way back above ground.

Example 1

The function that models the Screamer situation is a new parent function. To help you figure out what it is, sketch the right triangle shown in the diagram.

1. Sketch the right triangle shown in the diagram.

2. How does the height of the triangle relate to the escape heights you calculated in the Screamer problem?

b. Write an equation representing the escape height, e, for any operator, that is, for any angle of rotation of The Screamer. Note that the symbol \( \theta \) is the Greek letter “theta” and represents the measure of an angle.

c. On your graphing calculator, adjust the viewing window so that you can see all of the data. Then graph your equation from part (c) on top of the data. How well does your equation for the escape height fit your data from The Screamer problem?

d. Adjust the viewing window so that you can see more of your graph. Describe the behavior of the graph as \( \theta \) gets larger. Does this make sense? Why or why not?

e. Use the [TABLE] function of your calculator to see the values for your equation. Add another column to your table from The Screamer, label it with the equation you wrote for the escape height, and enter these values, rounded to the nearest hundredth. How do the values of your function compare with your measured data?
Lesson 9.1.2C Resource Page
Unit Circle↔Graph

(Based off of Core Connections Integrated III, Second Edition, Chapter 9.1.3 by Judy Kysh and Michael Kassarjian)

Subject and grade level: Secondary Math 3- Grades 10-12

Approximate time: 50 minutes

Rationale for Methods: This lesson follows an Algorithmic-Skill lesson where students are presented with a new algorithm used to solve a problem. Students will first be presented with a problem and an explanation of the purpose of the algorithm. Students will then predict outcomes of the algorithm and see the overview of the algorithm. The instructor will then walk students through a step-by-step explanation of the algorithm, and students will try it themselves. Lastly, any misconceptions or errors using the algorithm are corrected and students will polish their skill using a method called “overlearning” where students reuse this algorithm over a long period of time. This method is rooted in cognitive development research which shows that students gain a better understanding, not only of the algorithm, when this method is used, but they also are able to understand when to use it, predict outcomes, and therefore catch errors performed while conducting the algorithm.

Content Standards:

- F.IF.7e: Graph exponential and logarithmic functions showing intercepts and end behavior; and trigonometric functions, showing period, midline, and amplitude.
- F.TF.2: Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Academic Language/Vocabulary objectives:

- Language Skill: Students will need to recall the information they have learned about right triangles and angles. Students will then apply this knowledge to identify reference angles in the unit circle.
- Discipline-Specific Vocabulary: Students must learn what a reference angle is, and understand that there are triangles in each quadrant that correspond with a given reference angle. Students will need to remember the definition of a terminal side, quadrant, and theta.

Required material, resources, and technology:

- Smart Board or some way to project the “Unit Circle↔Graph” PowerPoint on the screen
- “Y=sin(θ) The Sine Calculator” worksheet

The PowerPoint provides an outline of the lesson and students can print them out before hand so they will have a kind of guided notes on which to write down their observations. The Smart
board allows for the instructor to record the students’ observations on the board and save them electronically for students to refer back to and also for students who were absent. The “Y=sin(θ) The Sine Calculator” worksheet gives students the individual opportunity to see how the sine graph connects to the unit circle. It also will give them experience finding angles which have the same reference angle.

**Lesson Objectives:** Students will develop an understanding of reference angles and will explore the connections between the sine graph and the unit circle (Standards F.IF.7e, and F.TF.2).

**Instructional Procedures:**

1. Project “Unit Circle↔Graph” PowerPoint on the electronic board/projector, and hand out “Y=sin(θ) The Sine Calculator” worksheet. Stand at front of classroom to cue to students the start of lecture and that they should pull out their notes.
2. Read Example 1 part a and b, then go to the next slide to show the circle. Draw a right triangle anywhere in the first quadrant of the circle and label its angle with θ (the Greek letter Theta), its height with y and its length with x.
3. Ask the students again if any other riders had to climb the same distance to get to safety? Common responses are: “Yeah, people just across to the left.” “And also if you were to mirror the triangles across the x-axis.”
4. Draw these triangles on the circle and label them completely.
5. Read part c of Example 1 to the students and discuss. Students are usually pretty quick to see that all the triangles are congruent and have the same angle between their terminal side and the x-axis. Be sure to emphasize that each one of the triangles is in a different Quadrant (just like there are different quadrants in the coordinate plane), and that the triangle in the first quadrant is called the reference angle, and can be used to help us find all this corresponding angles when just given one of them. They are also all the same distance from the x-axis thus making them look like a bowtie. This is a great way of checking to make sure that you have the right corresponding angles.
6. Show students the circle graph again with all of the triangles. Label 0°, 90°, 180°, 270°, and 360° on the circle. Tell students that when we are given an angle, and instructed to find the corresponding angles, we first need to calculate the reference angle. This can be found using our known angles: 0°, 90°, 180°, 270°, and 360°. If our given angle is in the first quadrant, then that is our reference angle θ. The three corresponding angles are then calculated by the formulas, 180° − θ (for the triangle in the second quadrant), 180° + θ (for the triangle in the third quadrant), and 360° − θ (for the triangle in the fourth quadrant). Emphasize this point by showing how we did the exact same things on the circle graph at the beginning of class just without the formalities.
7. Tell students that these formulas also work when you are given an angle not in the first quadrant, but with one small change. When you are given an angle not in the first quadrant students are to first, identify which quadrant it is in, then second, set the formula
that is associated with that quadrant equal to the given angle, then third solve for \( \theta \).

Once students identify \( \theta \), they can follow the same method they did when they were given \( \theta \).

8. As an example, tell students that they are given the angle 140°. Identify that 140° lies in the second quadrant therefore we set \( 180° - \theta = 140° \) and solve to get \( \theta = 40° \). To find the last two corresponding angles we take \( 180° + \theta \) which is 220° and \( 360° - \theta \) which is 320°.

9. Have students engage in the worksheet “Y=sin(\( \theta \)) The Sine Calculator.” As they work, walk around the classroom monitoring students and helping where needed. The worksheet will have students try the algorithm themselves first, by using the sine calculator to identify the angles, then by using the algorithm.

10. After the majority of the students have finished the worksheet, recollect the class and discuss the problems. Have a few select students come to the board to explain and draw their responses to the different prompts. While they do so, discuss any problems or misconceptions students had using the algorithm.

11. Assign the Learning Log and Assignment 9.1.3 to the students.

**Adaptations/Accommodations:** Printouts of the PowerPoint allows Student 1 to focus more on the class discussion and algorithm than getting distracted by copying notes. Having students work individually on the worksheet allows the instructor to walk around the classroom and monitor students work. Frequently sauntering by Student 1’s desk facilitates one-on-one discussion of his discoveries or to discreetly correct misconceptions. The class discussion after the worksheet gives Student 2 the opportunity to discuss her findings and to present them to the class by doing problems on the board. Doing the worksheet in class also helps Student 2 to have one less thing to worry about completing and possibly forgetting to turn in. The Learning Log also provides both Student 1 and Student 2 to elucidate, using words, the depth of their understanding of how the unit circle and sine function are connected.

**Assessment:** The worksheets will be collected at the end of the class period and looked over by the instructor. Students only received participation points for completing the worksheet as it is used as a formative assessment of what the students have learned about reference angles. The Learning logs also provide a formative assessment of how in depth students understand the connection between the unit circle and the sine graph. Students will continue to use this information in subsequent lessons, quizzes, assignments, and the end of unit test.
SECONDARY MATH 3
UNIT CIRCLE ↔ GRAPH

Objectives:
Students will develop an understanding of reference angles and will explore the connections between the sine graph and the unit circle.

OS: F.17a, F.17b

EXAMPLE 1
Given a circle centered at the origin, draw a triangle that could represent Roni and Anton's position on the ship when the ship made a sudden stop. a. Label the triangle with its height and its angle measure (from 0 degrees).
   b. Did any other triangles have to climb the same distance to get to a similar top or down? If yes, label the length and angle measure and label them completely.
   c. What is the relationship between these triangles? The angle in the first quadrant is called the reference angle. What is a method for determining the length of all of the other corresponding angles when you are given just one?

EXAMPLE 2
In Example 1, you used a unit circle to calculate the height of a seat on The Screamer. Can you use your graph of \( y = \sin(x) \) instead to determine the height?
   a. Use the sine calculator on your worksheet to determine the height of a seat that has rotated 150° from the starting point.
   b. Are there any other seats at exactly the same height? If so, indicate them on your sine calculator graph.
   c. Now, use the graph to calculate which angles correspond to seats with the same height?

LEARNING LOG
With your team, discuss the ways in which a unit circle and the graph of \( y = \sin(x) \) are connected. Record your ideas in your Learning Log. Use diagrams, arrows, and other math tools to facilitate your ideas. Label this entry "Unit Circle ↔ Graph for \( y = \sin(x) \)" and include today's date.

ASSIGNMENT
3.1.3 #20-30, 2-21, 2-31, 9-90
1. What is the height of a seat that has rotated 130° from the starting platform?

a. Are there any other seats at exactly the same distance? If so list them below with their corresponding angles and indicate them on the graph below.

2. In the small unit circle below, draw in each angle. Don’t forget to label the angles and their heights.
3. For each of the following angle measures, locate the other angle with the same corresponding height. Then sketch a small unit circle, draw in each pair of angles, and label their heights.
   a. $80^\circ$
   b. $200^\circ$
   c. $310^\circ$
1. What is the height of a seat that has rotated 130° from the starting platform?
   \[ \approx 0.77 = 77 \text{ feet} \]
   a. Are there any other seats at exactly the same distance? If so list them below with their corresponding angles and indicate them on the graph below.
   \[ 50°, 230°, \text{ and } 310° \]

2. In the small unit circle below, draw in each angle. Don’t forget to label the angles and their heights.

Heights = 77 feet
3. For each of the following angle measures, locate the other angles with the same corresponding height. Then sketch a small unit circle, draw in each pair of angles, and label their heights.
   a. $80^\circ$
   b. $200^\circ$
   c. $310^\circ$

   heights $= 99$ feet
   heights $= 35$ feet
   heights $= 78$ feet
Assignment 9.1.3

9-30. Sketch a graph of the first two cycles of \( y = \sin(\theta) \). Then label your graph to show the following positions of a passenger on The Screamer.

a. The passenger gets on initially.

b. The passenger reaches the bottom of the water pit.

c. The passenger is halfway between the highest point of The Screamer and the ground level.

9-31. The graph at right represents the height above ground of a rider on The Screamer. Draw a unit circle and mark each of the corresponding locations of the labeled points. Then describe where the rider is located at each point.

9-32. The graphs of \( y = \log_2(x - 1) \) and \( y = x^3 - 4x \) intersect at two points: (2, 0) and approximately (1.1187, −3.075). Use that information to determine the solutions to \( \log_2(x - 1) = x^3 - 4x \).

9-36. Planets-Are-Us has found a box company called We-Be-Cubes that only makes cubic boxes.

a. If Planets-Are-Us wants a box that will fit the Super Jupiter piñata, which is a sphere with a radius of 2 feet, what is the smallest box that will work? Give the dimensions and the volume.

b. We-Be-Cubes is having a clearance sale. Their Lucky 13 box, which has a volume of 13 cubic feet, is 50% off! What size piñatas can fit in the Lucky 13 box?

c. Now the Operations Director of Planets-Are-Us needs your help! Create a formula for calculating the radius (ft) of the largest spherical piñata that a box with volume \( v \) (cubic ft) can hold.
Assignment 9.1.3- KEY: Answers are in Bold

- **9-30.** Sketch a graph of the first two cycles of \( y = \sin(\theta) \). Then label your graph to show the following positions of a passenger on *The Screamer.* [See graph at right.]
  
  d. The passenger gets on initially.
  
  e. The passenger reaches the bottom of the water pit.
  
  f. The passenger is halfway between the highest point of *The Screamer* and the ground level.

- **9-31.** The graph at right represents the height above ground of a rider on *The Screamer.* Draw a unit circle and mark each of the corresponding locations of the labeled points. Then describe where the rider is located at each point. [(A): above ground just past the highest point, slightly left of center; (B): just below ground and left of center; (C): back to the starting point. See diagram at right.]

- **9-32.** The graphs of \( y = \log_2(x - 1) \) and \( y = x^3 - 4x \) intersect at two points: (2, 0) and approximately (1.1187, –3.075). Use that information to determine the solutions to \( \log_2(x - 1) = x^3 - 4x \). \([x = 2 \text{ or } x \approx 1.1187.]\)

- **9-36.** Planets-Are-Us has found a box company called We-Be-Cubes that only makes cubic boxes.
  
  d. If Planets-Are-Us wants a box that will fit the Super Jupiter piñata, which is a sphere with a radius of 2 feet, what is the smallest box that will work? Give the dimensions and the volume. \([\text{The box must be at least 4 ft by 4 ft by 4 ft for a volume of at least 64 cubic feet.}]\)
  
  e. We-Be-Cubes is having a clearance sale. Their Lucky 13 box, which has a volume of 13 cubic feet, is 50% off! What size piñatas can fit in the Lucky 13 box? \([\text{Piñatas with radius less than 1.18 feet, or about 14 inches.}]\)
  
  f. Now the Operations Director of Planets-Are-Us needs your help! Create a formula for calculating the radius (ft) of the largest spherical piñata that a box with volume \( v \) (cubic ft) can hold. \([r = \sqrt[3]{\frac{v}{2}}]\)
Student 1’s “$Y = \sin(\theta)$ The Sine Calculator” Worksheet (page 1)

Secondary Math 3: $Y = \sin(\theta)$ The Sine Calculator

Show all your work. Simplify your answers completely.

O WORK = NO CREDIT

Name: __________________________  Date: __________

1. What is the height of a seat that has rotated $130^\circ$ from the starting platform?

   a. Are there any other seats at exactly the same height? If so list them below with their corresponding angles and indicate them on the graph below.

   ![Graph of sine function]

2. In the small unit circle below, draw in each angle. Don’t forget to label the angles and their heights.

   ![Unit circle with angles drawn]
3. For each of the following angle measures, use the sine calculator from the resource page to determine the corresponding height and to locate another angle with the same corresponding height. Then sketch a small unit circle, draw in each pair of angles, and label their heights.

a. 80°
   \[ Y = \sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{2} \]
   height = 0.866

b. 200°
   \[ Y = \sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{2} \]
   height = 0.707

c. 310°
   \[ Y = \sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{2} \]
   height = 0.866
Secondary Math 3: \( Y = \sin(\theta) \) The Sine Calculator

Show all your work. Simplify your answers completely.

\( 0 \) WORK = NO CREDIT

1. What is the height of a seat that has rotated 130° from the starting platform?

\[ \sim 7.75 \]

a. Are there any other seats at exactly the same height? If so list them below with their corresponding angles and indicate them on the graph below.

2. In the small unit circle below, draw in each angle. Don’t forget to label the angles and their heights.
3. For each of the following angle measures, use the sine calculator from the resource page to determine the corresponding height and to locate another angle with the same corresponding height. Then sketch a small unit circle, draw in each pair of angles, and label their heights.

   a. 60°

   b. 200°

   c. 310°

   height: 0.99

   height: 0.35

   height: 0.75

   \[ \sin \theta = y \]

   \[ \cos \theta = x \]

   \[ (\cos \theta, \sin \theta) = (\cos \theta, \sin \theta) \]

   \[ r = \sqrt{1 - \sin^2 \theta} \]
Student 1’s Assignment 9.1.3
Student 2’s Assignment 9.1.3

30

Time 35min

A = At the top of the ride
    starting to go down

B = Just barely under ground
    level

C = Finished one rotation
    & back where you
    started

32

\log_2 (x-1) = x^2 + 4x

20 \:\: x = 2, x \approx 1.11, \: x^2

36

x^n \: V = 2 \cdot (n+1)

V = 4 \cdot (\pi \cdot r^2)

V = 16 \cdot \pi \cdot r^3

-10
“Unit Circle ↔ Graph for \( y = \sin(\theta) \)."

The unit circle and the Sin graph are connected in many ways as the circle goes around the graph goes up and down so where the circle is negative the graph is also negative if you put the points from the circle into a graph you will get a sin graph.

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**Student 2’s Learning Log**

On a unit circle and on a graph of \( \sin(\theta) \) there are 4 distinct points; highs lows and zeros. On these two graphs you can see how the two graphs line up in this way. X-intercepts are marked with light blue and highs and lows are marked with dark blue.
Graphing and Interpreting the Cosine Wave
(Based off of Core Connections Integrated III, Second Edition, Chapter 9.1.4 by Judy Kysh and Michael Kassarjian)

Subject and grade level: Secondary Math 3- Grades 10-12

Approximate time: 65 minutes

Rationale for Methods: This lesson follows a Discover-the-Relationship type lesson (also called inquiry-based learning) with the aid of a hands-on activity using a model of “The Screamer.” The Discover-the-Relationship method is rooted in research of cognitive development which has shown that as students discover a mathematical relationship themselves, they are: (1) more likely going to remember it, (2) create a firm understanding of that topic, and (3) make more sense of other things concerning that topic, for example algorithms.

Content Standards:
- F.IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F.IF.7e: Graph exponential and logarithmic functions showing intercepts and end behavior; and trigonometric functions, showing period, midline, and amplitude.
- F.TF.2: Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Academic Language/Vocabulary objectives:
- Language Skill: Students will again need to collect data using a model and record any observations they have during the activity. Students will analyze and discuss patterns and connections between the data they collected and its graph. Students will also need to recall their knowledge of trigonometry in relation to right triangles to discover the connection between the Unit Circle and the Cosine graph.
- Discipline-Specific Vocabulary: Students will learn the connection between the Unit Circle and the Cosine graph.

Required material, resources, and technology:
- Smart Board or some way to project the “Graphing and Interpreting the Cosine Wave via the Screamer” PowerPoint on the screen
- The Screamer Ferris Wheels from the “Graphing the Sine Wave via The Screamer” Lesson
- Graphing Calculators, one per pair.
The PowerPoint provides an outline of the lesson and students can print them out before hand so they will have a kind of guided notes on which to write down their observations. The Smart board allows for the instructor to record the students’ observations on the board and save them electronically for students to refer back to and also for students who were absent. The Screamer models allows for students to have a hands-on learning experience where they will discover how the unit circle relates to the cosine wave. Lastly, the graphing calculators allow for each student to then input their observations recorded from working with the The Screamer and graph them, thus helping them connect it to the cosine wave.

**Lesson Objectives:** Students will use the cosine function to calculate horizontal distances in a unit circle. They will draw conclusions about the relationship between the sine and cosine of an angle and the sine and cosine graphs (Standards F.IF.4, F.IF.7e, and F.TF.2).

**Instructional Procedures:**

1. Project “Graphing the Sine Wave via the Screamer” PowerPoint on the electronic board/projector. Stand at front of classroom to cue to students the start of lecture and that they should pull out their notes.
2. Begin the discussion by pairing the students up and having them Think-Pair-Share about how they would write the coordinates of point P on the second slide of the PowerPoint. Give the students about 3 minutes to do this. Walk around the classroom and monitor discussions while they do.
3. After the three minutes have pairs share what they discussed. Throughout the discussion be sure to emphasize that the coordinates of point P are \((x, y)\) and we have been using the sine function to calculate the height of P which corresponds with the value \(y\). Also be sure to emphasize that if we draw a right triangle using the terminal side of the angle as our hypotenuse, then the base of the triangle is our \(x\) value and can be found using the \(\cos(34^\circ)\).
4. On the next slide, show students that this method can be used when given any angle \(\theta\), and the coordinates of that point would be \((\cos \theta, \sin \theta)\).
5. On the next slide, ask students “If you know the sine of an angle in the unit circle, can you determine its cosine? How?” Have students get back with their partner and Think-Pair-Share again. This time only give them 2 minutes. Monitor while they do.
6. Bring the class back together and have pairs share. Since they are working with a right triangle, many of them will see that they can use the Pythagorean Theorem to solve for cosine, or in other words, the missing side. Show this algebraically on the board.
7. Now use this information to do the example on the next slide.
8. In order to help students discover why the cosine graph can be found using the unit circle. Have students, in pairs, use The Screamer Farris wheels to measure the base of each triangle created by the different positions of the seats. They will make a table on their own paper with the angle as the \(x\)-values and the corresponding lengths and the \(y\)-values. Once they have done this for at least 15 different seat positions, have them graph it in their graphing calculators.
9. Talk about student’s discoveries and observations. Be sure to point out how these numbers match with the numbers which make up the \( y = \cos(x) \) graph.

10. Do example 1 on the PowerPoint.

11. Assign Assignment 9.1.4

**Adaptations/Accommodations:** Printouts of the PowerPoint allows Student 1 to focus more on the class discussion than getting distracted by copying notes. Using The Screamer model not only helps students to be more engaged and using different learning mediums, but it also allows Student 1 to see how something new he is learning, for example trig functions and the unit circle, connect to things he already knows like trig functions in relation to right triangles. Having students work in groups allows the instructor to walk around the room and listen to discussions and help students in smaller groups. This is a great help to Student 1 who then was able to ask questions and work with peers to answer questions and it provided Student 2 with the opportunity to discuss her findings and think critically about why she is observing what she is. Class discussions also provide Student 2 with an opportunity to share these discoveries.

**Assessment:** Students use a program called MyOpenMath to take quizzes in class during subsequent lessons on which this material will be formatively assessed. Specific questions in the assignment are also used to not only help the instructor understand how well the students are understanding the material, but will also give the instructor the opportunity to give each student individual feedback and to let them know where they stand in relation to the objectives. At the end of the unit there will also be a test where students will need to use their knowledge of the unit circle, sine function, and other trig functions to solve problems.
Graphing and Interpreting the Cosine Function

Objectives:
- Graphing and interpreting the cosine function
- Understanding the unit circle and its relationship with the cosine function

1. Work with your team to write the coordinates of point $P$ on the unit circle shown. Is there more than one way to determine the coordinates of point $P$? Be prepared to share your strategies with the class.

2. New generalise what you found in the previous problem to write the coordinates of point $Q$ on the unit circle shown.

3. If you know the size of an angle in the unit circle, can you determine its cosine? How?

4. The angle $\theta$ in the unit circle at the right has a cosine of $\cos(\theta)$.
   a. What are the exact coordinates of point $Q$? Use the Pythagorean Theorem.
   b. What is the exact value of $\cos(\theta)$?

5. 9-42
   Obtain a copy of the Lesson 9.1A Resource Page. You will use the same process to graph the cosine function as you did to graph the sine function, but you will use the base of the triangle instead of the height. Why?
   a. Label the length of the base of each triangle in the unit circle. Plot these lengths at their corresponding angle locations on the graph. You will be plotting points in the form $(\theta, \cos(\theta))$.
   b. Draw five more triangles that are congruent to the given five triangles but are located in the second quadrant. Label the triangles with their angle measures (from $\theta$) and the length of the bases. Add five more points corresponding to these triangles to the graph.
**Example 1**

Do you remember The Screamer? LaRosa does! She was riding The Screamer sitting 27 horizontal feet away from the central support pole when the ride stopped. What was her seat’s angle of rotation? Is there more than one possibility? Justify your answer using as many representations as you can.

**Assignment**

9.1.4 49-65, 9-47, 9-57
Assignment 9.1.4

- **9-45.** What are the coordinates of points $P$ and $Q$ on the unit circle at right?

- **9-46.** The measure of $\angle ROS$ in $\Delta ROS$ below is $60^\circ$.
  
  a. The curved arrow represents the rotation of $OR$, beginning from the positive $x$-axis. Through how many degrees has $OR$ rotated?
  b. If $OR = 1$, what are the exact lengths of $OS$ and $SR$?
  c. What are the exact coordinates of point $R$?

- **9-47.** Determine the exact coordinates of a point on the unit circle that has $\sin(\theta) = \frac{1}{4}$.

- **9-57.** What is the equation of the graph at right?
Assignment 9.1.4- KEY: Answers in Bold

- **9-45.** What are the coordinates of points $P$ and $Q$ on the unit circle at right?
  
  $P: (\cos(50^\circ), \sin(50^\circ)) \text{ or } \approx (0.643, 0.766)$;
  
  $Q: (\cos(110^\circ), \sin(110^\circ)) \text{ or } \approx (-0.342, 0.940)$

- **9-46.** The measure of $\angle ROS$ in $\triangle ROS$ below is 60º.

  a. The curved arrow represents the rotation of $OR$, beginning from the positive $x$-axis. Through how many degrees has $OR$ rotated? $[300^\circ]$  
  
  b. If $OR = 1$, what are the exact lengths of $OS$ and $SR$?  
  
  $[\frac{1}{2} \text{ and } \frac{\sqrt{3}}{2}]$  
  
  c. What are the exact coordinates of point $R$?  
  
  $[\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)]$

- **9-47.** Determine the exact coordinates of a point on the unit circle that has $\sin(\theta) = \frac{1}{4}$.
  
  $[\left(\frac{\sqrt{15}}{4}, \frac{1}{4}\right) \text{ or } \left(-\frac{\sqrt{15}}{4}, \frac{1}{4}\right)]$

- **9-57.** What is the equation of the graph at right?
  
  $[y = -\frac{1}{4}(x - 2)(x + 2)^2]$
Student 1’s Assignment 9.1.4

4.5, 4.6, 4.7, 97

4.5

1.a) \( \cos(10), \sin(10) \)

\( P(\cos(50), \sin(50)) \)

\( Q(0.847, 0.539) \)

\( P(0.642, 0.787, 60.565) \)

4.6

A \( 60^\circ \)

B \( 90^\circ \)

a) \( 360^\circ - m \angle PAB \)

\( m \angle PAB = 30^\circ \)

\( m \angle PAB = 30^\circ \)

b) \( \cos(30) = 0.86603 \)

4.7

\( y = (x+2)^2 (x-2) \)

2. \( z = 2 (2)^2 (-2) \)

\( z = -16 \)

\( \frac{z}{m} = \frac{-16}{m} = 1 \)

\( y = \frac{1}{4} (x+2)^2 (x-2) \)
Student 2’s Assignment 9.1.4

Time: 15 min

\[ x = -\frac{2y^2}{y} \]

\[ a \left( x + 2 \right)^2 \left( x - 2 \right) = y \]

\[ a \left( 0 + 2 \right)^2 \left( 0 - 2 \right) = 2 \]

\[ a \left( y - 2 \right) = \frac{y}{4} \]

\[ x + 2 \]

\[ \left( x + 2 \right)^3 \left( x - 2 \right) = 2 \]

Your assignment is really good, redo it next time if you spill...
Reflection

Analyze student learning

During the first lesson where students were measuring The Screamer to find the “heights” of the seats, Student 1 was not only able to see the patterns of repeating numbers, but was also able to predict how the graph would look when we plotted the numbers. His understanding of this can be seen in the student’s learning log entitled “Student 1’s Learning Log” included with Unit Circle↔Graph Lesson. Student 1 was also able to recall his previous knowledge of sine graphs to be able to identify what type of graph it was after we graphed it and participated well in the class discussion.

During the Y=sin(θ) The Sine Calculator Worksheet, Student 1 was able to easily identify which angles/triangles had the same reference angle (see Student 1’s Y=sin(θ) The Sine Calculator Worksheet). However, his understanding of how to calculate the angles’ measures was extremely lacking, and this became more prevalent as time went on. During daily quizzes there were questions where students were given a measure of an angle and the value of one of its trig values. The student was then prompted to identify its reference angle and other trig values. Student 1 was consistently incorrectly identifying the reference angle and therefore was unable to set up the triangles correctly in order to identify the other trig values. Because of this, I was able to give Student 1 some one-on-one instruction to try and better help him understand. Student 1 became a little more proficient as he was now able to correctly graph the given angle measures, but still had a hard time understanding what formula to use when calculating the reference angle.

Discussing the connection between the cosine graph and the unit circle during the Graphing and Interpreting the Cosine Wave lesson really seemed to resonate with Student 1 just
like it did when we were talking about the Sine Wave in the first lesson. Student 1 was able to identify patterns in the cosine values and how they were represented in its graph. This understanding, along with the understanding of how the sine function relates to the unit circle, gave Student 1 the tools he needed to understand how we can write the coordinates of a point on the unit circle using sine and cosine functions. This can be seen in Student 1’s Assignment 9.1.4.

Student 2 also provided great insight during class discussions. During the Graphing the Sine Wave via the Screamer lesson, Student 2 was able to quickly identify patterns and how they would be represented in the graph. Some of these patterns she identified are expressed in her Learning Log entitled “Student 2’s Learning Log.”

When learning about reference angles, I noticed that once Student 2 had finished her worksheet, she was helping others around her to understand how to know which formula to use to calculate the measures of the angles. I was surprised when during the first daily quiz (in which she was given an angle measure and prompted to identify its reference angle) that she was unable to correctly identify the reference angle. However, after talking with Student 2 about her thought process when doing the daily quiz, I observed that Student 2 did not graph the angle measure first and therefore incorrectly identified what quadrant she was working in, which therefore messed up her calculations of the reference angle. This made sense, because during the Y=\sin(\theta) The Sine Calculator Worksheet Student 2 was first prompted to graph all the angles with the same reference angle and then calculate the angle measures. However, when she was just given one angle measure and prompted to identify its reference angle, student 2 did not see the importance of identifying exactly which quadrant you are working in so as to make sure of the positive and negative values. Being able to quickly identify this misconception helped give Student 2 the tools she needed to better understand how to write the coordinates of a point on the
Analyze teaching effectiveness

Overall I am very happy with these lessons. The discussions we had were extremely beneficial and helped students, especially Student 1, to understand the bigger picture of how the Unit Circle connects to the sine and cosine graphs. They also provided Student 2 with the opportunity to share her insights and time to critically think about the reasons behind the patterns she was observing. Sometimes these discussions ran a little longer than expected but I found that they were the most beneficial to helping students understand the reasoning behind the mathematics, so when we finally go to it, they understood why we were doing what we were doing.

In the first lesson plan I originally planned to have students graph the values they measured on The Screamer using their graphing calculators. However, the first section I taught this lesson, while trying to explain to students how to enter in the numbers and then correctly graph it in the graphing calculators, I had a significant amount of students who were struggling to know what button to press next, where they needed to plug in their numbers, what page they needed to get to in order to select each data set, and so on and so on. This struggle with the technical aspect of the graphing calculators was taking away from the actual objective of the lesson, so I decided to just graph the points on my computer, using an online program called Desmos.com, and was also then able to graph the sine graph right on top of the data points students collected. This was such a better method as it allowed students to see how the points they measured directly correlated with the values of the sine function, instead of getting caught up in the technical aspect of graphing them on the graphing calculators.
If I were to go back and modify this unit plan I would definitely go back to the Unit Circle↔Graph lesson and do more example problems with the students. Overall, the Y=sin(θ) The Sine Calculator Worksheet was very successful at helping students to identify all the angles that have the same reference angle, but how the problems were presented did not have enough example noise, that is, there were not enough different types of examples that required students to find the reference angle of any given angle. In the Y=sin(θ) The Sine Calculator Worksheet the examples were all the same, however, the students were never presented with problems just like these ever again in the unit. They were given problems where they were given one angle measure and needed to find the reference angle in radians or degrees, or they were given a reference angle and instructed to find the angle measure of the angle in a certain quadrant that also had this reference angle. These are the types of problems that if I were able to go back, I would connect the material taught in the Y=sin(θ) The Sine Calculator Worksheet and build upon it and show students the types of problems where what they learned in the worksheet was applicable. I believe this would have helped out Student 1 significantly as he would have been able to see the material used in many different ways, and it would have helped Student 2 to understand the importance of identifying what quadrant you are in first and would have therefore eliminated that misconception before it even became one.

In the future I would also have a quick review of trig in relation to right triangles before the unit started. Several of the students were able to remember which trig functions related to which sides on the triangles, but most students did not. Having a review first would have helped students not only better understand why we used cosine and sine for the measures we did, but also how to write the coordinates of a point on a Unit Circle, and how to calculate the other trig functions of an angle when you are given one of them.
Overall, I found this a great learning experience. It allowed me the opportunity to put into action the skills and methods I have learned throughout my courses and provided me with the opportunity to really focus on adjusting material and lessons based on students’ needs. There is a wide range of abilities in a single classroom and all students are going to need something a little different to challenge them and make them grow.